

Neutrino Physics

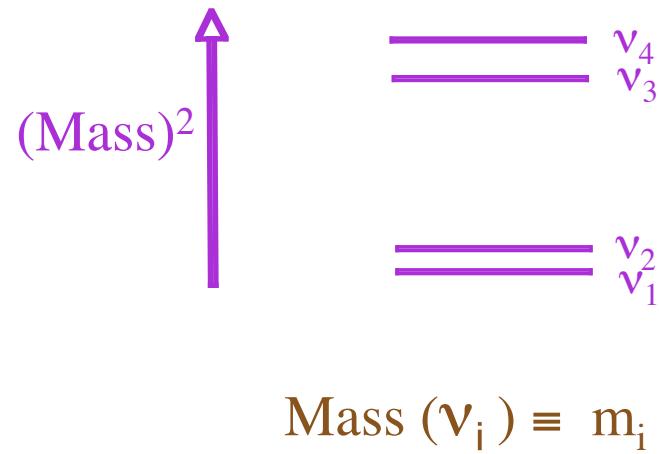
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Fermilab
March 24, 2005

Breakthrough —

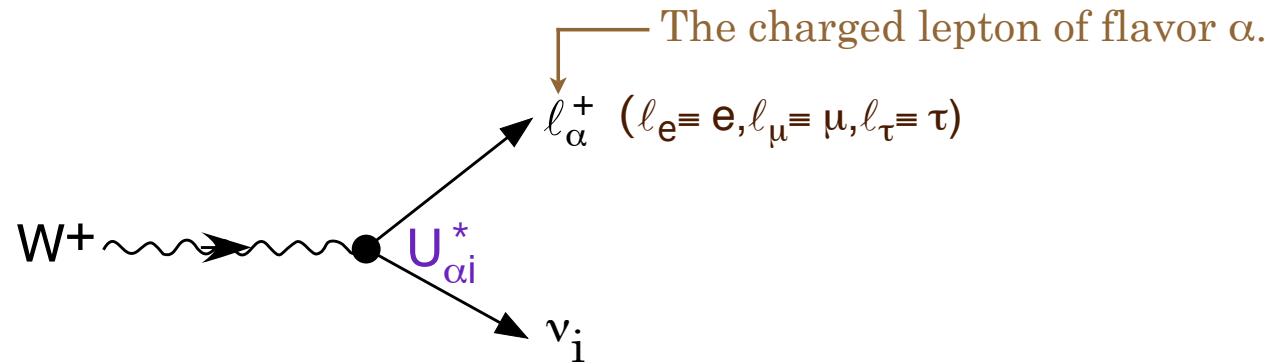
Neutrinos have nonzero masses!

Leptons mix!

There is some spectrum of 3 or more neutrino mass eigenstates ν_i :



Mixing means that in —



a given ℓ_α^+ can be accompanied by *any* ν_i .

The neutrino state emitted together with ℓ_α^+ is

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle .$$

↑
The neutrino of flavor α .

U is the Leptonic Mixing Matrix. U is unitary.

Each mass eigenstate is a superposition of flavors:

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle$$

The flavor- α fraction of ν_i is —

$$|\langle \nu_{\alpha} | \nu_i \rangle|^2 = |U_{\alpha i}|^2$$

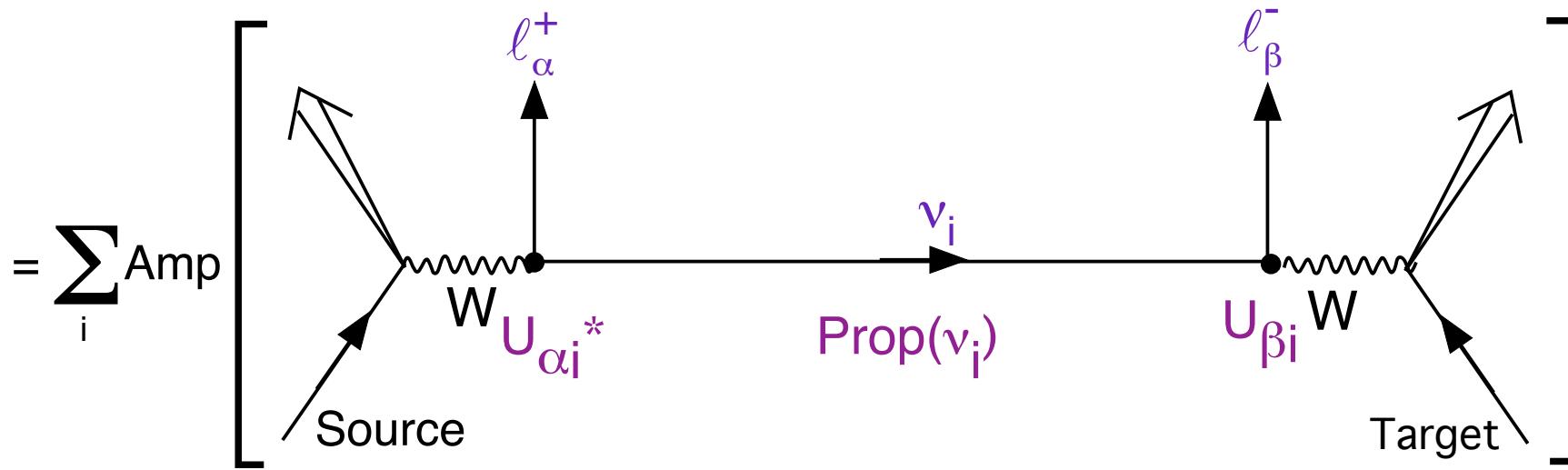
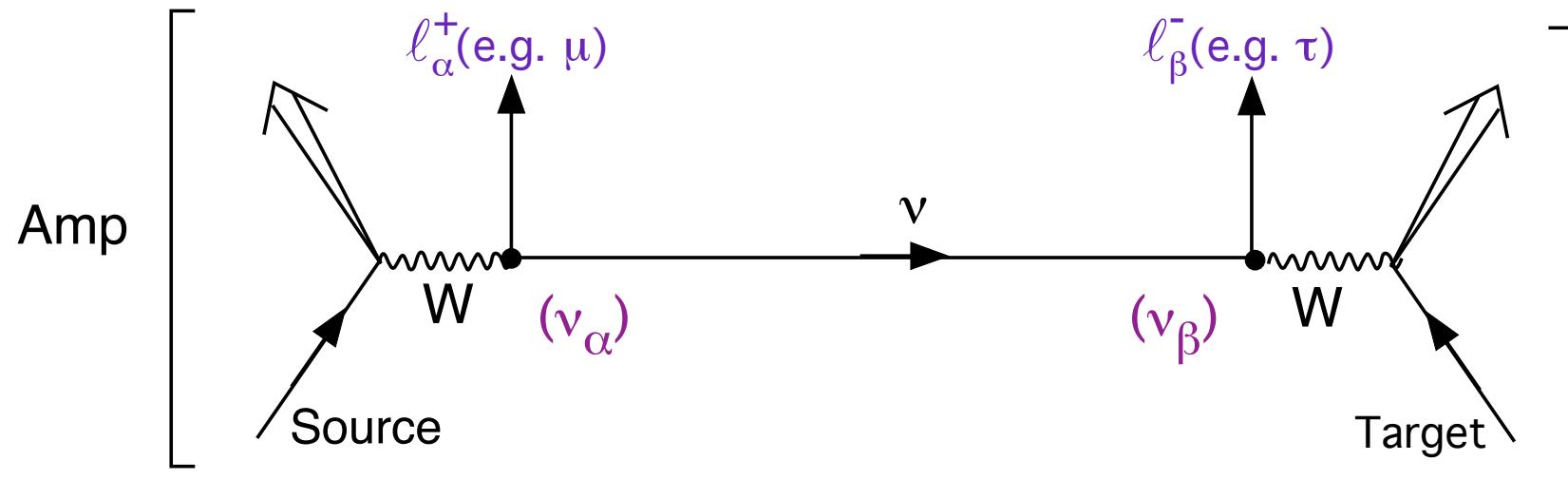
The Evidence for Neutrino Mass:

Neutrino Flavor Change

Neutrino Flavor Change (Oscillation)

in Vacuum

(Approach of
B.K. & Stodolsky)



$$\text{Amp } [v_\alpha \rightarrow v_\beta] = \sum U_{\alpha i}^* \text{Prop}(v_i) U_{\beta i}$$

What is Propagator (v_i) \equiv $\text{Prop}(v_i)$?

In the v_i rest frame, where the proper time is τ_i ,

$$i \frac{\partial}{\partial \tau_i} |\nu_i(\tau_i)\rangle = m_i |\nu_i(\tau_i)\rangle \quad .$$

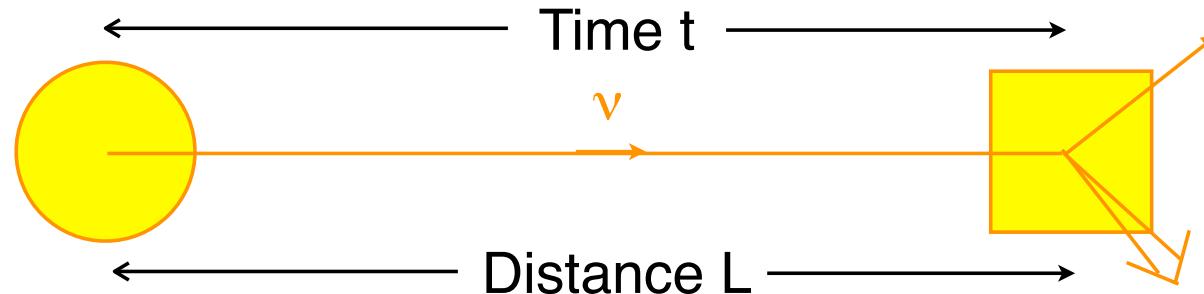
Thus,

$$|\nu_i(\tau_i)\rangle = e^{-im_i\tau_i} |\nu_i(0)\rangle \quad .$$

Then, the amplitude for propagation for time τ_i
is —

$$\text{Prop}(\nu_i) \equiv \langle \nu_i(0) | \nu_i(\tau_i) \rangle = e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t .

They are common to all components of the beam.

For each v_i , by Lorentz invariance,

$$m_i \tau_i = E_i t - p_i L .$$

Neutrino sources are \sim constant in time.

Averaged over time, the

$$e^{-iE_1 t} - e^{-iE_2 t} \quad \text{interference}$$

is —

$$\langle e^{-i(E_1-E_2)t} \rangle_t = 0$$

unless $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent. (Stodolsky)

For each mass eigenstate ,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} .$$

Then the phase in the v_i propagator $\exp[-im_i\tau_i]$ is —

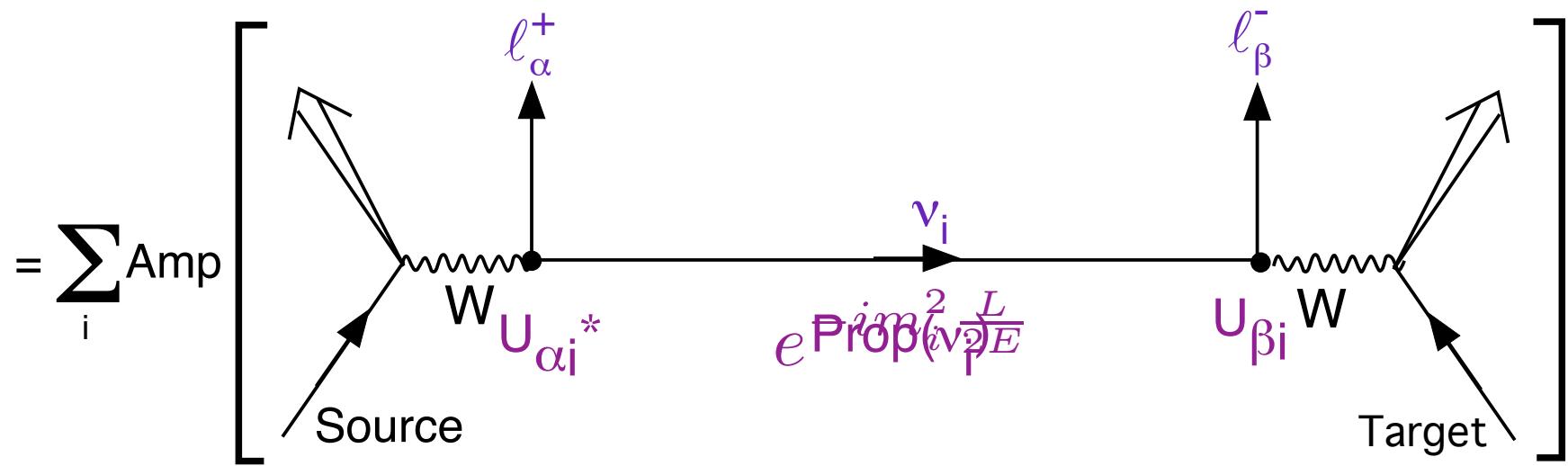
$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= E(t - L) + m_i^2 L / 2E .$$



Irrelevant overall phase

Amp $[\nu_\alpha \rightarrow \nu_\beta]$



$$= \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$+ 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E})$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$

For Antineutrinos —

$$P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \stackrel{CPT}{=} P(\nu_\beta \rightarrow \nu_\alpha) \stackrel{Lastslide}{=} P(\nu_\alpha \rightarrow \nu_\beta; U \rightarrow U^*)$$

Thus,

$$P(\overset{\leftarrow}{\nu}_\alpha \rightarrow \overset{\leftarrow}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$+ 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E})$$

A complex U would lead to the CP violation

$$P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta) .$$

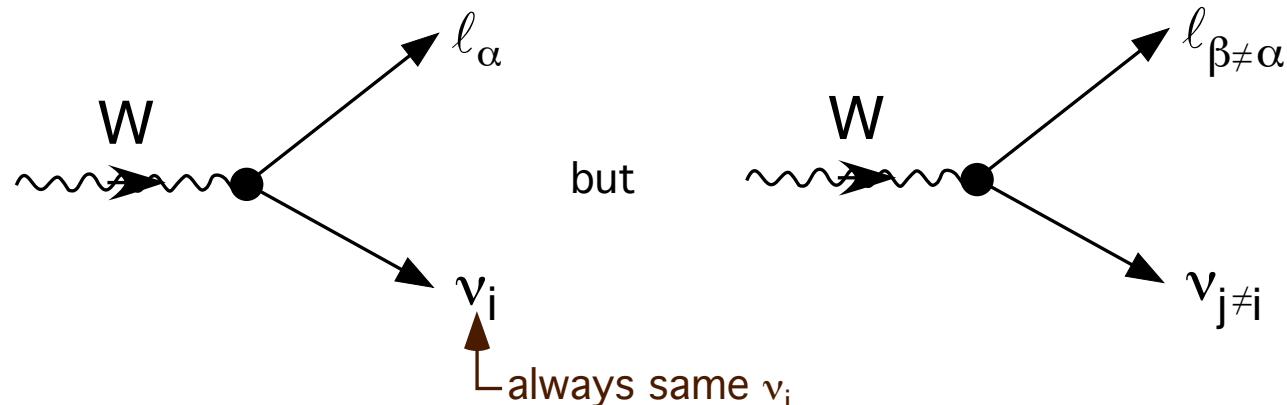
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P(\overleftarrow{\nu}_\alpha \rightarrow \overleftarrow{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* \Rightarrow ν Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta \neq \alpha, i} = 0, \text{ so that } P(\overleftarrow{\nu}_\alpha \rightarrow \overleftarrow{\nu}_\beta) = \delta_{\alpha\beta}.$$

Flavor *change* \Rightarrow Mixing

3. One can detect ($\nu_\alpha \rightarrow \nu_\beta$) in two ways:

See $\nu_{\beta \neq \alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including \hbar and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})}$$

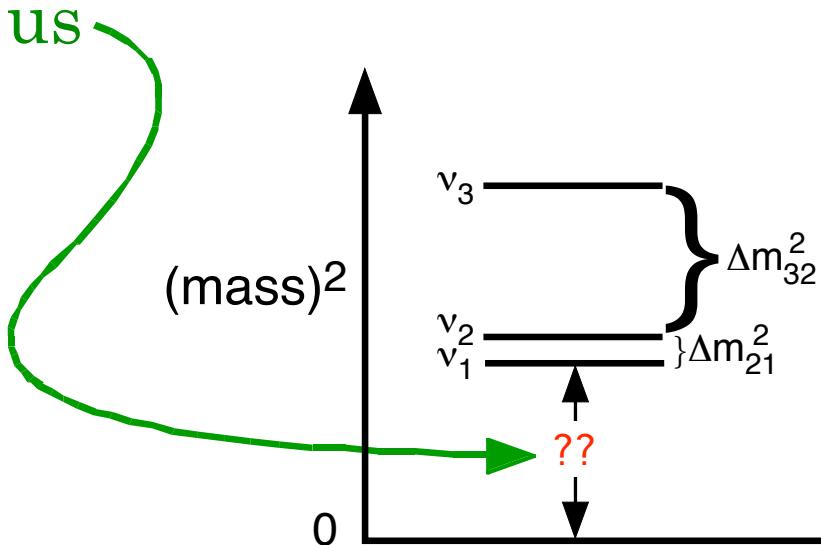
$\sin^2[1.27 \Delta m^2 (\text{eV})^2 \frac{L(\text{km})}{E(\text{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2 (\text{eV}^2) \gtrsim \frac{E(\text{GeV})}{L(\text{km})} .$$

5. Flavor change in vacuum oscillates with L/E.
Hence the name “neutrino oscillation”. {The
L/E is from the proper time τ .}

6. $P(\vec{\nu}_\alpha \rightarrow \vec{\nu}_\beta)$ depends only on squared-mass
splittings. Oscillation experiments cannot
tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\vec{\nu}_\alpha \rightarrow \vec{\nu}_\beta) = 1$$

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the *active* flux disappears:

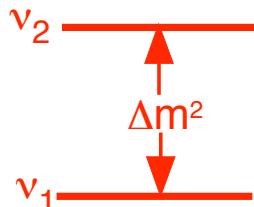
$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

8. Assuming all coherent ν_i in a beam have a common **momentum p**, rather than a common energy E, is a harmless error.

This assumption leads to the same $P(\overset{\leftrightarrow}{\nu}_\alpha \rightarrow \overset{\leftrightarrow}{\nu}_\beta)$.

When Only Two Neutrinos Count

This is frequently the case.



The diagram shows two horizontal red lines representing neutrino mass states. The top line is labeled ν_2 and the bottom line is labeled ν_1 . Between them is a vertical double-headed arrow labeled Δm^2 , indicating the mass difference between the two states.

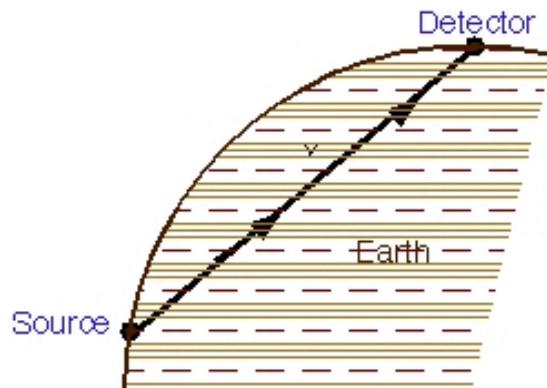
$$U = \begin{bmatrix} \nu_e & \nu_1 & \nu_2 \\ \nu_\mu & \cos \theta & \sin \theta \\ & -\sin \theta & \cos \theta \end{bmatrix} ; \quad \begin{aligned} \nu_e &= \nu_1 \cos \theta + \nu_2 \sin \theta \\ \nu_\mu &= \nu_1 (-\sin \theta) + \nu_2 \cos \theta \end{aligned}$$

Mixing angle

$$P(\overleftarrow{\nu}_e \leftrightarrow \overleftarrow{\nu}_\mu) = \sin^2 2\theta \sin^2 [1.27 \Delta m^2 (\text{eV}^2) \frac{L(\text{km})}{E(\text{GeV})}]$$

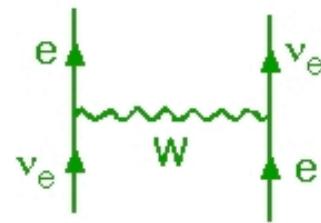
$$P(\overleftarrow{\nu}_{e,\mu} \rightarrow \overleftarrow{\nu}_{e,\mu}) = 1 - \quad \text{“} \quad \text{“}$$

Neutrino Flavor Change in Matter



Coherent forward scattering from ambient matter can have a big effect.

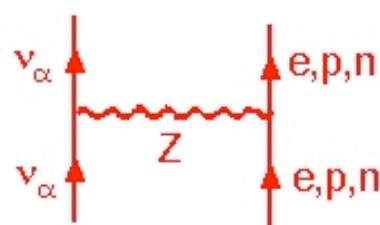
Interaction



Interaction Potential Energy

$$V_W = +\sqrt{2}G_F N_e \quad (- \text{ for } \bar{\nu}_e)$$

#e/vol



$$V_Z = -\frac{\sqrt{2}}{2}G_F N_n \quad (+ \text{ for } \bar{\nu}_\alpha)$$

#n/vol

Neutrino propagation in matter is conveniently treated via a Schrödinger Equation:

$$i \frac{\partial}{\partial t} \nu(t) = H \nu(t)$$

Matrix in flavor space
Multi-component
in flavor space

To illustrate, we describe the case —

When Only Two Neutrinos Count

$$\nu(t) = \begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix} ; \quad H = \begin{bmatrix} \nu_e & \nu_\mu \end{bmatrix} \begin{bmatrix} \nu_e & \nu_\mu \\ \nu_\mu & \nu_\mu \end{bmatrix}$$

Amp. to be a ν_e
 ↓
 $f_e(t)$
 ↑
 Amp. to be a ν_μ

A 2x2 matrix in
 ν_e - ν_μ space

$$i \frac{\partial}{\partial t} \begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix} = \begin{bmatrix} & H \\ & \end{bmatrix} \begin{bmatrix} f_e(t) \\ f_\mu(t) \end{bmatrix}$$

In Vacuum:

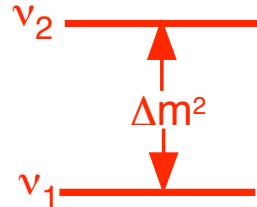
$$\langle \nu_\alpha | H | \nu_\beta \rangle = \langle \sum_i U_{\alpha i}^* \nu_i | H | \sum_j U_{\beta j}^* \nu_j \rangle = \sum_j U_{\alpha j} U_{\beta j}^* \sqrt{p^2 + m_j^2}$$

↑
Momentum of the beam

In flavor change, only **relative** phases, hence **relative** energies, matter.

∴ In H, any multiple of the Identity Matrix I may be omitted.

In Vacuum



$$U = \begin{bmatrix} \nu_e & \nu_\mu \\ \nu_\mu & \nu_e \end{bmatrix} \begin{bmatrix} \nu_1 & \nu_2 \\ \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} ; \quad \begin{aligned} \nu_e &= \nu_1 \cos \theta + \nu_2 \sin \theta \\ \nu_\mu &= \nu_1 (-\sin \theta) + \nu_2 \cos \theta \end{aligned}$$

It follows that, omitting a piece $\propto I$,

$$H_{\text{Vac}} = \frac{\Delta m^2}{4E} \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} .$$

With Schrödinger's Equation, this gives the usual $P(\nu_e \rightarrow \nu_\mu)$.

The eigenvalues of H_{Vac} are —

$$\pm \frac{\Delta m^2}{4E} \equiv \pm \lambda .$$

With $c \equiv \cos \theta$, $s \equiv \sin \theta$,

$$\nu_e = \nu_1 c + \nu_2 s \xrightarrow{t} \nu(t) = \nu_1 c e^{i\lambda t} + \nu_2 s e^{-i\lambda t}$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= |<\nu_\mu|\nu(t)>|^2 = |sc(-e^{i\lambda t} + e^{-i\lambda t})|^2 \\ &= \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}) \end{aligned}$$

In Matter

$$H_M = H_{\text{Vac}} + V_W \begin{bmatrix} \nu_e & \nu_\mu \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} + V_Z \underbrace{\begin{bmatrix} \nu_e & \nu_\mu \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix}}_{\propto I, \text{ so drop}}$$

$$H_M = H_{\text{Vac}} + \frac{V_W}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \underbrace{\frac{V_W}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{}$$

$$H_M = \frac{\Delta m^2}{4E} \begin{bmatrix} -(\cos 2\theta - x) & \sin 2\theta \\ \sin 2\theta & \cos 2\theta - x \end{bmatrix},$$

with $x \equiv \frac{V_W/2}{\Delta m^2/4E} = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2}$.

The Effective Splitting and Mixing in Matter

If we define —

$$\Delta m_M^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - x)^2}$$

and

$$\sin^2 2\theta_M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2} ,$$

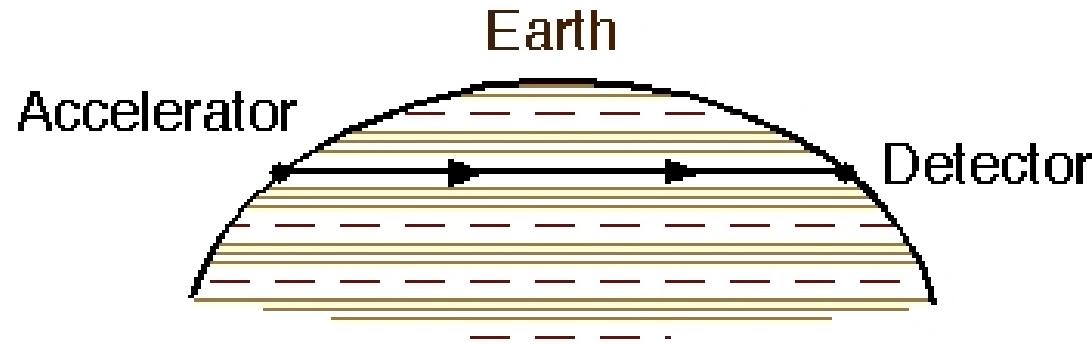
then

$$H_M = \frac{\Delta m_M^2}{4E} \begin{bmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{bmatrix} .$$

This is H_{Vac} with $(\Delta m^2, \theta) \rightarrow (\Delta m_M^2, \theta_M)$.

Thus, Δm_M^2 and θ_M are the effective splitting and mixing angle in matter.

Travel Through the Earth



The matter density encountered en route is \sim constant.

Thus, H_M is position-independent, just like H_{Vac} .

Therefore, in the earth (but not too deep),

$$P_M(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta_M \sin^2(\Delta m_M^2 \frac{L}{4E})$$

↑
In matter

The Size and Consequence of the Matter Effect

The matter effect depends on —

$$x = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2} \propto E .$$

The denominator contains a Sign

In the earth's mantle, for $|\Delta m^2| = |\Delta m^2(\text{atmospheric})| \approx 2.5 \times 10^{-3} \text{ eV}^2$,

$$|x| \simeq \frac{E}{12 \text{ GeV}} .$$

Since $V_W(\bar{v}) = -V_W(v)$, $x(\bar{v}) = -x(v)$.

Thus $\overline{\Delta m_M}^2 \neq \Delta m_M^2$ and $\sin^2 2\bar{\theta}_M \neq \sin^2 2\theta_M$.

The matter effect causes an asymmetry between \bar{v} and v oscillation. This must be separated from the genuine CP asymmetry.

The MSW Effect

Since —

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - x)^2} ,$$

even a tiny vacuum mixing $\sin^2 2\theta$ can be amplified into a near-maximal in-matter mixing $\sin^2 2\theta_M$ if

$$x \cong \cos 2\theta .$$

This is the “resonant” version of the —

Mikheyev Smirnov Wolfenstein Effect.